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THE PARALLAXES OF STARS AS DERIVED FROM SPECTRAL CLASS AND APPARENT MAGNITUDE

By Knut Lundmark

The founder of stellar astronomy, Sir William Herschel, introduced in his pioneer work about the Construction of the Heavens the hypothesis that the stars have practically the same actual brightness or luminosity. F. G. W. Struve, who discussed and extended Herschel's work about the constitution of the stellar universe in his famous work *Etudes d'Astronomie Stellaire* (1847), also assumed the stars to have more or less the same luminosity. But after this time astronomers started with a new hypothesis: the stars were considered to have luminosities distributed according to the law of error. From this point of view Gylden, in the year 1872, derived relations between the mean parallax and the number of stars for the different magnitudes. Later, this problem has been investigated from different points of view by Seeliger, Kapteyn, Schwarzchild, Charlier and others.

In parallel with this theoretical development it was sought to obtain data on the luminosities of the stars from micrometric determinations of their parallaxes. As is well known, many troubles arose in the early determinations of those minute quantities and the first parallax data could not give any clear indication as to the distribution of stellar luminosities. When spectrum analysis of the stars had made its victorious advances it was suspected that different classes of stars have different luminosities, as the spectral classes are closely related to the effective temperatures. Nothing definite was established until Hertzsprung pointed out that among the yellow stars there are enormous differences in absolute brightness and that two main subdivisions according to absolute magnitude exist among these stars. The names *giants* and *dwarfs* were proposed by Hertzsprung and these have since been generally accepted. In the year 1914 Russell showed, in his address before the American Astronomical Society, that the existence of giants and dwarfs could be well traced for the spectral classes G and K but that the absolute magnitude had a rather constant value for B and A stars. The discussion of these and other observational facts led Russell to establish his well known theory of stellar evolution. The extensive and high-standard work by Adams and his colaborators at Mount Wilson with regard to spectroscopic parallaxes and Pease's measures of stellar diameters seem to establish beyond doubt the existence of giant and dwarf stars.

The admirable work at the Allegheny, Leander McCormick, Yerkes, Mount Wilson, Sproul, Dearborn and Greenwich observatories in trigonometrical determinations of stellar parallaxes has furnished extremely valuable material for our knowledge of stellar luminosities. It is a rather sensational fact that the number of parallaxes determined at Allegheny under Dr. Schlesinger's and Dr. Curtis's supervision has exceeded 600, and the very extensive work of the other parallax observatorics is likewise held in high esteem.

In Russell's above mentioned address it was pointed out that a rather close correlation exists between the absolute magnitude and the spectral class for the dwarf branch. A formula $M = a + 2.1\ (\mathrm{Sp} - A)$ where M is the absolute magnitude and Sp denotes the spectral class (B = 1, A = 2, etc.) was suggested. The numerical value of the constant a was found to be 1.4 for stars of directly measured parallaxes and 0.6 for stars in moving clusters.

Russell computed the probable error in the determination of absolute magnitude from his formula and the actually-determined values, by assuming the residuals to represent the errors, and found the probable error for cluster stars to be $\mp 0^{m}.61$ and for other stars $\mp 0^{m}.94$. Even in this last case it will be possible by assuming a constant value for the absolute magnitude of the stars in each of the different spectral classes merely from a knowledge of the spectrum and the giant or dwarf character to predict the parallaxes with an uncertainty of only 43 per cent.

The existence of new and very valuable trigonometric parallaxes suggests immediately a re-discussion of the relations between spectra and absolute magnitude. The existing parallax material was compiled and reduced to the system given by Dr. van Maanen and Miss Wolfe in *Mount Wilson Contributions* No. 189. The question now arises whether the star parallaxes may be affected by unknown systematic errors due to imperfect knowledge of the mean parallax for the comparison stars or errors of instrumental origin. Van Maanen and Miss Wolfe's discussion shows that this can not be the case as the derived systematic corrections for most of the modern observers are very small. In deriving these systematic corrections use was made of formulæ for the mean parallax which in turn depends on measured parallaxes. Thus the possibility remained that slight systematic errors (of the second order) could exist in the system given by those authors. Recently Stromberg has determined the systematic errors for different series of parallaxes from another point of view. The fact that the systematic and accidential errors of spectroscopic parallaxes are proportional to the parallaxes themselves, while the errors in trigonometric parallaxes are independent of the values, made it possible for him to determine from a comparison of the two systems the true systematic error for each system or each observer. The derived systematic corrections show an excellent agreement with those found by van Maanen and Miss Wolfe. When we remember that the results are obtained by using entirely different methods it is evident that no unknown systematic error exists in the star parallaxes when they are reduced to van Maanen's and Miss Wolfe's system.

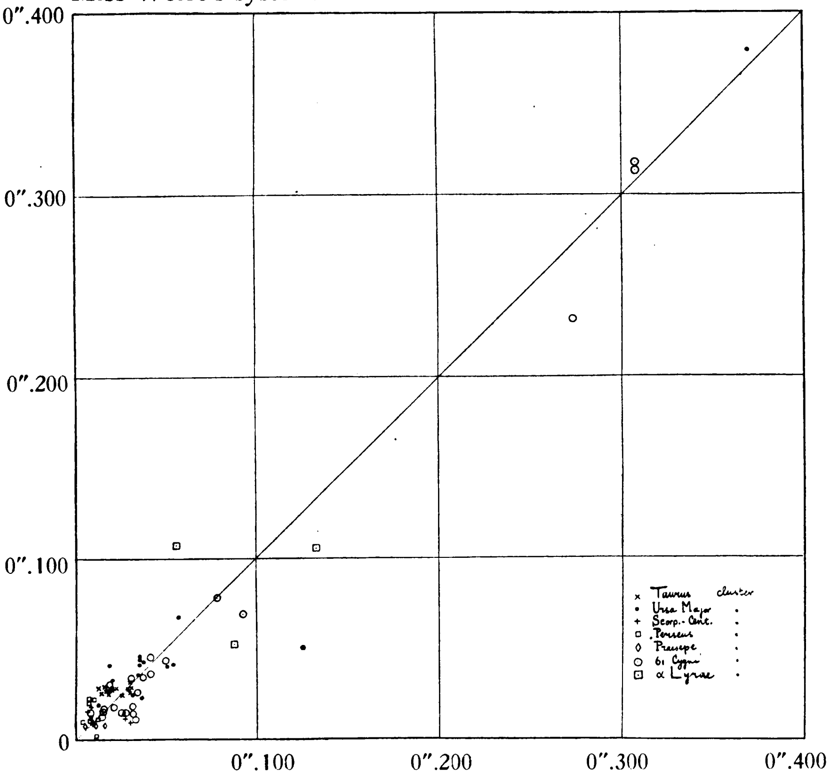

Figure 1

Figure 1—Ordinates are parallaxes from moving clusters. Abscissae are absolute parallaxes.

As a small contribution to the question of systematic errors in measured parallaxes, may be mentioned a comparison made by the writer between 84 members of moving clusters for which trigonometric or spectroscopic parallaxes also are available. The parallaxes for the members in moving clusters are entirely independent of direct measures as they are derived from elements depending only upon the motion. The comparison, as illustrated in Figure 1, shows that no sensible systematic difference exists between the two series of parallaxes. It will be admitted that the material is not very extensive, but parallel values from $0''.376$ to $0''.005$ are represented in this comparison and even for small values of the parallax no systematic errors seem to be present.

The parallax material has been used to study the distribution of absolute stellar magnitudes[1]. It is evident that the use of the new material strengthens Russell's conclusions. In the dwarf branch of Russell's diagram it has been found that the probable error of the absolute magnitude is in the mean $\mp 0^m.6$. which permits a prediction of the parallax with an uncertainty of 30 per cent. As a comparison it may be mentioned that the uncertainty in the Mount Wilson spectroscopic parallaxes is estimated by Adams to be 18 per cent. For some classes of giant stars and for the classes A to Fo the range in absolute magnitude is sufficiently small that a fair parallax value can be computed from the assumed mean absolute magnitude.

From the discussion of the mean absolute magnitude as derived from trigonometric parallaxes, moving cluster parallaxes and spectroscopic and hypothetical parallaxes the following preliminary formulæ are derived (m is the apparent magnitude):

Spectral Class	Luminosity Class	Spectral Parallax
B5		$\log \pi = -0.2m - 1.10$
A0		$-0.2m - 0.76$
A2		$-0.2m - 0.70$
A5		$-0.2m - 0.56$
F0		$-0.2m - 0.50$
F2	dwarfs	$-0.2m - 0.42$
F5	"	$-0.2m - 0.30$
F8	"	$-0.2m - 0.16$
G0	giants	$-0.2m - 1.30$
G0	dwarfs	$-0.2m - 0.10$
G5	giants	$-0.2m - 0.88$
G5	dwarfs	$-0.2m - 0.04$
K0	giants	$-0.2m - 0.79$

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[1] Compare also Dr. Curtis' paper in *Publ. A. S. P.*, **34**, 33, 1922, and *Mount Wilson Report*, 1921, p. 274.

K0	dwarfs	— 0.2m + 0.24
K5	giants	— 0.2m — 0.90
K5	dwarfs	— 0.2m + 0.44
Ma	giants	— 0.2m — 0.92
Ma	dwarfs	— 0.2m + 1.04
Md	super giants	— 0.2m — 1.56
O	"	— 0.2m — 1.80
N	"	— 0.2m — 1.26
Cepheids . .	"	— 0.2m — 1.53
Novae . . .	"	— 0.2m — 2.40

The values for Md and O stars are derived from proper motions and radial velocities. The mean absolute magnitude of N stars has been kindly communicated by Dr. Moore, as resulting from his extensive investigations of the radial velocities of these stars. For Cepheids the mean absolute magnitude is taken from Adams spectroscopic parallaxes. The space distribution of the novœ has been investigated by the writer using different methods and the mean absolute magnitude used here is a preliminary result from this work.

For Md stars, Cepheids, novœ, and in case of variability N stars the *maximum magnitude* should be used in applying the formulæ for computing parallaxes.

It has not been considered essential to derive a linear formula for the relation between spectral class and dwarf absolute magnitude as this can not be done without giving up the equal intervals between different spectral classes in the Harvard system, the use of which adds to the convenience of the present method.

For using the formulæ given in this paper it must be known whether a star, with a given spectrum, belongs to the giant or the dwarf branch. If we do not want to know the degree of absolute magnitude, but only whether the object is a giant or a dwarf several criteria can be used. The best will, of course, be that of the spectral lines used by Adams at Mount Wilson. For spectrograms obtained with objective prism Lindblad has suggested a criterion for luminosity by using the "cyanogen" bands. Besides a knowledge of the proper motion will in many cases furnish a valuable test for the giant or dwarf character. The curves found by Luyten for the relation between reduced proper motion ($m + 5 + 5 \log \mu$) and absolute magnitude will be very useful in this connection. The radial velocity correlated with the absolute magnitude can also be used for a discrimination between the giant and dwarf classes. Another criterion seems to proceed from Albrecht's discovery of a correlation of wave-length with spectral type and absolute magnitude.

For the early classes of stars the absolute magnitude seems to be rather constant and no division of the absolute magnitude into different groups corresponding to giants and dwarfs is indicated. For the earliest B stars the existence of "super-giants" may be suggested but the indications are very uncertain. Some of the A stars with sharp spectral lines may also form a group of "super-giants" but most of the stars in that class have an absolute magnitudes placing them in the ordinary giant class. For stars between F0 and G2 it has been recognized that the absolute magnitudes brighter than zero are rare and easy to discover from the appearance of the spectral lines, and the relative intensities of the hydrogen lines (*c*-character, according to Miss Maury's terminology). Considering the Ma stars it is evident that the dwarfs are very few. Only about 30 are known, and new objects of this class will be found from their large proper motions, as an Ma dwarf with an apparent magnitude of 10^{m} has a parallax of $0''.100$ and thus is one of the nearest stars. Attention is called to the usefulness of the formula above for Ma giants for obtaining a fair value of the parallax. The 2600 stars in the *Henry Draper Catalogue* with Ma spectra could, according to what has been said, safely be assumed to be giants and thus their distribution in space with rather good accuracy be determined.

As is well known, the O, B and N stars generally have a high and rather constant absolute magnitude. The Md stars are absolutely very bright at maximum brightness and belong more or less to the dwarfs at minimum brightness. The novœ have an extremely high absolute brightness at maximum. Their original or final absolute magnitude is sometimes high enough to place them, even in these evolutionary stages, in the giant class.

Thus, the need of criteria for separating giants and dwarfs is practically limited to G5—K8. Here we can have help from the color of a star in estimating its absolute magnitude. Some discoveries of Miss Maury as confronted with Russell's theory show that the color index for some of the spectral classes is slightly dependent on the absolute magnitude. The work of Seares, Lindblad and others have made this connection more clear and definite. It has been hoped that this color effect would give a means for deriving the absolute magnitude and hence the parallax, from determinations of the color alone; but as shown by Seares this is not possible. Used, however, in connection with spectral class an accurately determined color index may give us indications of the absolute magnitude. The data given by Seares in *Publ. A. S. P.*, **34**, 56, 1922, show that the change in color index for one magnitude in the spectral classes G5—K5 is in the mean $0^{m}.074$. It is probable that other color equivalents can also be adapted to that purpose, provided standard scales are established and some existing idiosyncrasies explained.

It is hoped that more accurate data will enable us to derive the parallax from the spectrum with still higher accuracy than 30 per cent. The differences in absolute magnitude for the same spectral class are due to the following causes: (1) A real range in absolute magnitude proceeding from differences in dimensions, even for stars with the same temperature, and from differences in temperature for stars classified as having the same spectrum; (2) A range arising from errors in the data used for determining absolute magnitude such as errors in measured parallaxes, errors in spectral classification and errors in photometric magnitudes. Sometimes also the use of the combined magnitude of unknown binaries will cause an error in the derived absolute magnitude.

Regarding (1) it can be assumed that the range is not larger than indicated by the probable error of $\mp$ $0^{m}.4$ in the absolute magnitudes measured at Mount Wilson. Trumpler's investigations of the physical members of the *Pleiades* cluster show the existence of a typical dwarf branch among the fainter stars in this cluster. Available spectra and color-indices, as determined by Shapley and Miss Richmond combined with Trumpler's data suggest a range in absolute magnitude for these stars corresponding to a probable error of $\mp$ $0^{m}.3$.

Regarding (2), the most important source of error for the present is the uncertainty in the spectral classification. It is estimated that a probable error of $\mp$ $0^{m}.3$ in M arises from the uncertainties in the Henry Draper spectra. As those are not established for purposes of determining absolute magnitude and parallaxes it must be said that the accuracy in the Harvard classification is very satisfactory.

The errors in M arising from inaccurate photometric data are small for the brighter stars but may amount for the fainter stars to $\mp$ $0^{m}.2$ or $\mp$ $0^{m}.3$.

The uncertainty in M due to errors in measured parallaxes is considerable if small trigonometric parallaxes are used, as the errors in direct parallax measures are independent of the value of the parallax itself. By using large or very accurately determined trigonometric parallaxes the resulting probable error in the absolute magnitude from inaccurate parallax data will be smaller than the probable error arising from accidental errors in the spectral data.

The use of Mount Wilson spectra seems to reduce the probable error in the mean absolute magnitude to ∓ 0″.5. Thus when the spectral class and the apparent magnitude are accurately known and reliable indications as to the giant or dwarf character are obtainable it seems possible to estimate parallaxes with a probable error of 23 per cent of the parallax value.

In the following table parallaxes are computed from the formulæ for some well known objects. The trigonometric and spectroscopic parallaxes are given in the fifth, sixth and seventh columns.

For spectral classes not used in Henry Draper the values of the mean absolute magnitude have been obtained by interpolation.

Name		Vis. Mag.	Spectral Class	Trigonometric Parallax	Spectroscopic Parallax Adams	Spectroscopic Parallax Shapley	Spectral Parallax
	h m			″	″	″	″
Groombr. 34	0 12.5	7.98	Ma	0.280	0.288		0.278
54 *Piscium*	0 34.2	6.08	K0	0.113	0.111	0.138	0.100
η Cassiopeiae	0 43.0	3.7	F8	0.184	0.132		0.126
		7.4	K5		0.151		0.055
τ Ceti	1 39.4	3.65	G7	0.323	0.316	0.316	0.234
ε Eridani	3 28.2	3.81	K1	0.321	0.275	0.275	0.315
o_2 “	4 10.7	4.5	K1	0.195	0.219	0.302	0.229
Sirius	6 40.7	—1.58	A0	0.372	0.376		0.360
Procyon	7 34.1	0.5	F3	0.310	0.347		0.380
22 *H. Camel.*	10 57.9	7.6	Ma	0.409	0.380		0.330
Arcturus	14 11.1	0.2	K0	0.105	0.158	0.209	0.158
α Centauri	14 32.8	0.33	G6	0.755	0.794		0.940
		1.70	K4		0.794		1.000
ξ Bootis	14 46.8	4.8	G6	0.179	0.158		0.126
		6.8	K4		0.105		0.100
70 *Ophiuchi*	18 0.4	4.3	K0	0.192	0.200	0.200	0.229
		6.0	K4		0.174		0.145
Vega	18 33.5	0.14	A	0.134			0.163
Altair	19 45.9	0.89	A5	0.214			0.183
61 *Cygni*	21 2.4	5.6	K7	0.306	0.347		0.302
		6.3	K8		0.302		0.288
ε Indi	21 55.7	4.74	K5	0.274			0.283

It may seem unnecessary to use this "short method" as spectroscopic parallaxes are determined on a large scale at Mount Wilson and work is started at other observatories. Of course, the parallaxes computed from spectra by the method here outlined will not in any way compete with measured parallaxes. These will always give the solid foundation for our knowledge of the distances of the stars. But for an extension of this knowledge to the fainter stars the use of the fact that the absolute magnitudes seem to group around certain mean values with a rather small dispersion may give better information about space distribution than statistical methods where all spectral classes are treated together. As the labor of establishing a system of reduction curves is not necessary for deciding whether a star belongs to the giants or the dwarfs, and as other than spectral criteria may be used for that purpose the determination of what we shall call the *spectral parallax* may be of value for utilizing some existing collections of stellar spectra where for some reason or another the material is not very well adapted for accurate measures of the absolute magnitude such as needed in applying Adams' method.

The following example will perhaps show a proper case for using the spectral-parallax method. The observers who possess spectrograms of the stars in the southern sky would like to use them for obtaining spectroscopic parallaxes. The plates are in general not strictly comparable with northern plates as different dispersion and different arrangements have been used in securing them. There is a woeful lack of trigonometric parallaxes in the southern sky and on that account serious trouble would arise in trying to establish the reduction curves required for applying the Mount Wilson method. Neither can moving cluster parallaxes or hypothetical parallaxes give enough data for that purpose. If available spectral material could be accurately classified and due attention given to the giant or dwarf characteristics, such a survey could in a short time furnish us with spectral parallaxes for the southern stars and considerably advance our knowledge of the space distribution of the nearest stars.

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